1.2a 

\[ y'' + y' - 2y = 0 \]

\[ y(t) = e^t \Rightarrow y' = e^t, \quad y'' = e^t \]

\[ e^t + e^t - 2e^t = 0 \quad \checkmark \quad \text{yes} \]

\[ y(t) = \sin t \Rightarrow y' = \cos t, \quad y'' = -\sin t \]

\[-\sin t + \cos t + 2\sin^2 t = 0 \quad \times \quad \text{no} \]

1.3)

(a) \[ w(t) = Ce^{4t} \]

(b) \[ 2g' = g \]
\[ g' = \frac{3}{2}g \]
\[ g(t) = Ce^{-\frac{1}{2}t} \]

(c) \[ p(t) = Ce^{-1t} \]
1.4 (guess & check) let

(a) \( y = ce^{2t} \)

\[ y' = 2ce^{2t}, \quad y'' = 4ce^{2t} \]

\[ y'' - 2y' + 2y = 0 \]

\[ 4ce^{2t} - 2(2ce^{2t}) - 2(ce^{2t}) = 0 \quad \checkmark \]

(b) \( y = ce^{-2t} \)

\[ y' = -2ce^{-2t} \]

\[ y'' = 4ce^{-2t} \]

\[ 2ce^{-2t} - 2(ce^{-2t}) = 0 \quad \checkmark \]

(c) \( y = ce^{-2t} \)

\[ y' = -2ce^{-2t}, \quad y'' = 4ce^{-2t} \]

\[ 4ce^{-2t} - 2(2ce^{-2t}) + 4(ce^{-2t}) = 0 \quad \checkmark \]

1.6, \( u = t^n, \quad u' = nt^{n-1} \)

\[ 2tu' = u \]

\[ 2nt^{n-1} = u \]

\[ 2nt^{n-2} = t^n \]

\[ 2n = 1 \quad \text{(n = 1/2)} \]
1.18. \( y'' = -y \quad y = \sin t, \quad y' = \cos t, \quad y'' = -\sin t \)

& \quad -\sin t = -\sin t \checkmark

(y = \cos t also works)
#1

**A**: large predators, small prey
- a small increase in y really decreases \( \frac{dx}{dt} \), predator eats a lot!
- a small increase in x doesn't help predator much, because prey is so small

**B**: large prey, small predator
- same logic just reversed

##2

\[
\frac{dy}{dt} = f(x, y)
\]

\( x \)

\( y \)

\( -2 \)

\( 0 \)

\( 2 \)

\( 8 \)

(a) \( y(t) \) is increasing when \( \frac{dy}{dt} > 0 \)
- \( y > 8 \) or \(-2 < y < 0\)

(b) \( y(t) \) is decreasing when \( \frac{dy}{dt} < 0 \)
- \( y < -2 \) or \( 0 < y < 8 \)

(c) \( \frac{dy}{dt} = 0 \) when \( y = -2, 0, 8 \)
- means \( y(t) \) is constant (equilibrium)
it shows when $y$ is increasing, decreasing and constant, equilibrium.

Also gives (relative) information about how quickly $y$ is changing (magnitude of $dy/dt$).

1. $L=0$:
   \[
   \left. \frac{dL}{dt} \right|_{t=0} = 0.5(1-0) = 0.5 = \left. \gamma \right|_{t=0}
   \]

2. $L=\gamma$:
   \[
   \left. \frac{dL}{dt} \right|_{t=\gamma} = 0.5(1-\gamma) = \left. \gamma \right|_{t=\gamma}
   \]

* Person with nothing learns faster at $t=0$.

3. $L=1$:
   \[
   \left. \frac{dL}{dt} \right|_{t=1} = 0
   \]
   Knowledge doesn't change! (no forgetting)

4. $L$ should approach 1, but never get there (horizontal asymptote)
   \[
   \frac{L}{t}
   \]