1. (a) Consider again the crop duster plane problem but this time the red mark slowly drifts toward the center as the propellers rotate as the plane rolls along the runway. Sketch what the four observers see this time.

(b) What do the four observers ideally see if the propellers are not rotating and the red mark drifts toward the center at a rate proportional to its distance from the center as the plane rolls along the runway?

2. Consider the same system of differential equations from problem 1. Use the GeoGebra applet https://ggbm.at/U3U6MsyA to generate predictions for the future number of rabbits and foxes if at time 0 we initially have the following different initial conditions: (i) 2 rabbits and 3 foxes, (ii) 1.5 rabbits and 4 foxes, and (iii) 4 rabbits and 2 foxes. For each of the different views, graph all three solutions on the same set of axes.

3. (a) Referring back to the rabbit and fox system of differential equations, suppose the current number of rabbits is 0 and the number of foxes is 2. Without using any technology and without making any calculations, what does the system of rate of change equations predict for the future number of rabbits and foxes? Explain your reasoning.

\[
\frac{dR}{dt} = 3R - 1.4RF \\
\frac{dF}{dt} = -F + 0.8RF
\]

(b) Use the GeoGebra applet to generate the 3D plot and all three different views or projections of the 3D plot. Show each graph and explain how each illustrates your conclusion in problem 3a).

(c) Suppose the current number of rabbits is 0 and the number of foxes is 6. What does the system of rate of change equations predict for the future number of rabbits and foxes? How and why is this prediction related to the prediction when the initial number of rabbits is 0 and the number of foxes is 2?
4. In previous problems dealing with two species, one of the animals was the predator and the other was the prey. In this problem we study systems of rate of change equations designed to inform us about the future populations for two species that are either competitive (that is both species are harmed by interaction) or cooperative (that is both species benefit from interaction).

(a) Which system of rate of change equations describes a situation where the two species compete and which system describes cooperative species? Explain your reasoning.

\[
\begin{align*}
\frac{dx}{dt} &= -5x + 2xy \\
\frac{dy}{dt} &= -4y + 3xy \\
\end{align*}
\]

\[
\begin{align*}
\frac{dx}{dt} &= 3x(1 - \frac{x}{3}) - \frac{1}{10}xy \\
\frac{dy}{dt} &= 2y(1 - \frac{y}{10}) - \frac{1}{5}xy \\
\end{align*}
\]

(b) For system (A), plot all nullclines and use this plot to determine all equilibrium solutions. Verify your equilibrium solutions algebraically.

(c) Use your results from (b) to sketch in the long-term behavior of solutions with initial conditions anywhere in the first quadrant of the phase plane. For example, describe the long-term behavior of solutions if the initial condition is in such and such region of the first quadrant. Provide a sketch of your analysis in the $x$-$y$ plane and write a paragraph summarizing your conclusions and any conjectures that you have about the long-term outcome for the two populations depending on the initial conditions.
5. Provide sketches of $x$ vs $t$ and $y$ vs $t$ for each of the following phase planes and solution curves.

(a) 

(b) 

(c) 

(d)